Turbulent Mixing Coefficients for Compressible Coaxial Submerged and Coflowing Jets

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This paper presents mixing coefficient correlations for turbulent, compressible, axisymmetric jets. Correlations are based upon 189 experiments from 36 investigations. Separate correlations are given for turbulent mixing coefficients for free jets without secondary flow and for coflowing jets with parallel, secondary flow. Velocity profile data and temperature/concentration data were correlated separately. Average percent differences in the mixing coefficient deduced from measurements and that predicted by the correlations were 11.7% and 15.2% for velocity data and temperature/concentration data, respectively.

Nomenclature

K	= turbulent mixing coefficient
M_5	= Mach number at half velocity point
M	= mass flux of particles, g/cm^2 sec
m	= decay slope (i.e., $d\ln U'/d\ln X$)
Pr^t	= turbulent Prandtl number
R	= reduced radial coordinate (r/r_1)
r	= radial coordinate, m
r_1	= primary jet radius, m
Sc^t	= turbulent Schmidt number
T'	= normalized temperature, $(T-T_\infty)/(T_1-T_\infty)$
T	= temperature, °K
x	= axial distance
X	= reduced axial coordinate (x/r_1)
X^*	= core length (x^*/r_1)
U	= dimensionless velocity (u/u_1)
U'	= normalized velocity, $(u-u_\infty)/(u_1-u_\infty)$
u	= axial velocity, m/sec
W	= mass fraction
ϵ	= eddy diffusivity, m^2/sec
ρ	= gas density, g/m^3

Subscripts

∞	= freestream/secondary stream
1	= primary jet
5	= half velocity point
c	= centerline
c	= concentration
i	= inner mixing boundary
o	= without secondary flow
s	= secondary
t	= turbulent or temperature
v	= velocity

Introduction

SEVERAL investigators have correlated or analyzed turbulent jet mixing data. The work of Harsha[1] was pertinent in this regard. Much of the earlier work was reviewed previously by the author and co-workers.[2] The recent work of Witze[3] provides an accurate method for predicting centerline decay of submerged jets, but cannot be applied to jets with secondary flows. Emphasis presently is being focused upon turbulent kinetic energy techniques for predicting jet structure.[4-6] This general method shows significant promise for predicting properties of turbulent jets. However, prediction of jet structure using the turbulent mixing coefficient concept (e.g., Ref. 7) still is applied widely to analysis of practical jets, such as rocket exhausts and entrained coal jets. Even so, mixing coefficients being used for coflowing jet predictions generally have not been developed for such applications and this has led to considerable error in jet predictions.

The work of Donaldson and Gray[7] has provided a basis for the work reported herein. Using the eddy diffusivity concept for turbulent mixing, these workers developed a model for predicting the mean properties (velocity, temperature, etc.) of compressible, turbulent freejets in the absence of secondary flow (i.e., submerged jets, see Fig. 1a). Using this model, together with jet mixing data for velocity, they deduced a mixing coefficient and showed that this coefficient was a function of the jet nozzle configuration and the half-velocity Mach number (M_5).

Smoot and Purcell[8] and Smoot et al.[9] extended this model solution to account for parallel, secondary flow of free and confined jets, respectively (i.e., coflowing jets, Fig. 1b). These models incorporated Prandtl's mixing length formulation for the velocity dependence of eddy diffusivity (see Ref. 10), wherein the rate of turbulent mixing is proportional to the difference in the velocities of the primary and secondary flows:

$$\epsilon_v/r_1u_1 = (K_v/2)\ (R_5 - R_i)\ |U_c - U_\infty| \tag{1}$$

$$\epsilon_c/r_1u_1 = (K_c/2)\ (R_5 - R_i)\ |U_c - U_\infty| \tag{2}$$

This formulation is being used extensively in turbulent jet mixing models. In order to evaluate the accuracy of this model, Tufts and Smoot[11] and Stowell and Smoot[2] obtained data from the literature for free and confined, nonreacting jets with secondary flows and compared results with model predictions, using the turbulent mixing coefficient of Donaldson and Gray for K_v which was deduced from data without secondary flows. Agreement was very poor, the average error in measured and predicted jet core lengths being well in excess of 200%. These investigators then recorrelated the mixing coefficient for coflowing, coaxial jets, and improved the predictive accuracy for jet core length to approximately 30% average error for a large number of jet experiments. The later work of Stowell and Smoot[2] was based upon much more data and also provided a simpler coorelation. However, this correlation was not as accurate as the original work of Donaldson and Gray for the submerged jets.

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Index categories: Jets, Wakes and Viscid-Inviscid Flow Interactions; Multiphase Flows.

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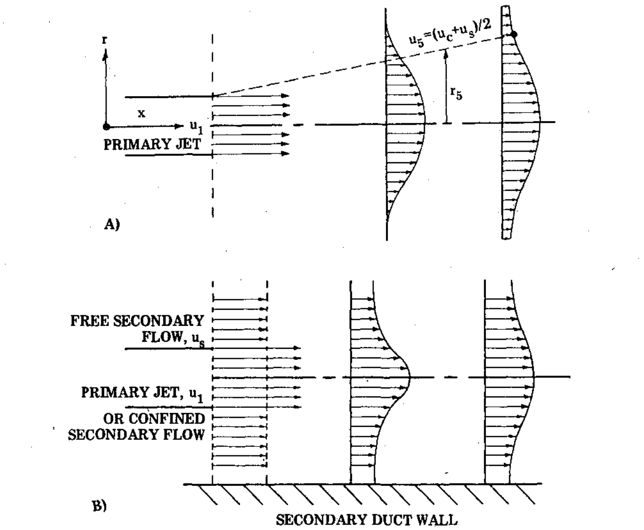

Fig. 1 Schematic diagram of coaxial jet configurations: A) Free jet without secondary flow (submerged jet); B) Free or confined jets with parallel secondary flow (co-flowing jets).

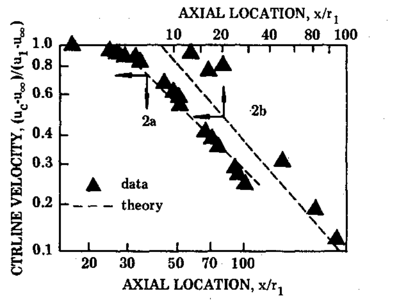

Fig. 2 Comparison of predicted and measured centerline velocity data (see Table 1 for test conditions).

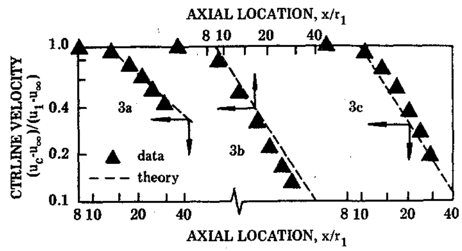

Fig. 3 Comparison of predicted and measured centerline velocity data (see Table 1 for test conditions).

The objective of this paper is to present the newly obtained correlations for coaxial, submerged and coflowing jets, and to discuss their accuracy and limitations. The jet mixing data have been recorrelated in order to correct the deficiency for submerged jets and to improve overall accuracy. These correlations can be used with available computer models to predict mean properties of jet structure for submerged and coflowing coaxial jets with generally known accuracy.

General Correlative Approach

There have been a large number of models proposed to describe the mean turbulent shear stress or the eddy diffusivity of coflowing jets.[7,8,10,12-15] All require a parameter, often referred to as the mixing coefficient K. In this study, rather than propose a new form for the mean turbulent stress or the eddy diffusivity, Eqs. (1) and (2) have been used and the resulting mixing coefficients deduced from each experiment then have been correlated for submerged and coflowing jets.

It has been observed experimentally for a wide class of jets that the decay of measured property (i.e., velocity, temperature, concentration) along the centerline of the jet is linear when plotted logarithmically. Figures 2-4 show such experimental data for several different jets. Table 1 summarizes the conditions and parameters for the data shown in Figs. 2-4. For free, submerged jets, the logarithmic velocity decay (d ln u_c/d ln x) can be shown explicitly to be linear and essentially constant, with a value near unity for all test conditions. This result is shown by differentiating the explicit velocity expression given by Witze.[3] For coflowing, free coaxial jets, an explicit expression from Smoot and Purcell[8] has been obtained which also yields a linear logarithmic decay. Further, using this same treatment, Stowell and Smoot[2] have shown theoretically for coflowing, free, coaxial jets that the logarithmic decay rate is independent of the turbulent mixing coefficient, when the value of mixing coefficient is the same in the core and developed regions.

Although no explicit equations for the decay rate are available for particle-laden jets, nonparallel jets, or reacting jets, the experimental evidence of Figs. 4 and 5 suggests that this logarithmic linearity is a very general observation, at least over a major portion of the centerline decay. Smoot and Purcell[8] also showed explicitly (for coflowing, free jets) that the core length of a jet is inversely proportional to the mixing coefficient. This same observation has also been demonstrated theoretically for more complex jets using numerical solutions.

The treatment of data used in this study has therefore been based on the following conditions: 1) the logarithmic centerline decay is linear, over a major part of the region, and 2) the core length is inversely proportional to the mixing coefficient. Consequently, the experimental centerline decay rate of concentration, temperature, or velocity in logarithmic coordinates, has been fit statistically and then this decay slope (m) has been extrapolated to a U_c', T_c', or W_c/W_l value of 1.0 to determine the core length (X^*), as illustrated in Figs. 2-

Table 1 Summary of conditions and parameters for data shown in Figs. 1 and 2

Figure	Ref.	System	Property	u_1/u_∞(m/s)	ρ_1/ρ_∞	Reaction	X*	m
2a	16	air/air	velocity	527/0	1.34	no	30.1	-1.11
2b	17	He/air	velocity	69/14	0.91	no	16.5	-1.21
2c	18	air/air	velocity	129/59	0.88	no	15.4	-0.89
3a	19	H_2/air	velocity	762/234	0.13	no	8.6	-1.28
3b	19	H_2/air	velocity	1006/159	0.088	no	11.7	-1.43
4a	20	H_2/air	velocity	151/15 107/21	.069	yes	5	-0.91
4b	20	H_2/air	hydrogen element	151/15 107/21	.069	yes	5	-0.91
4c	21	He/air	temperature	61/31	0.83	no	19.2	-1.1

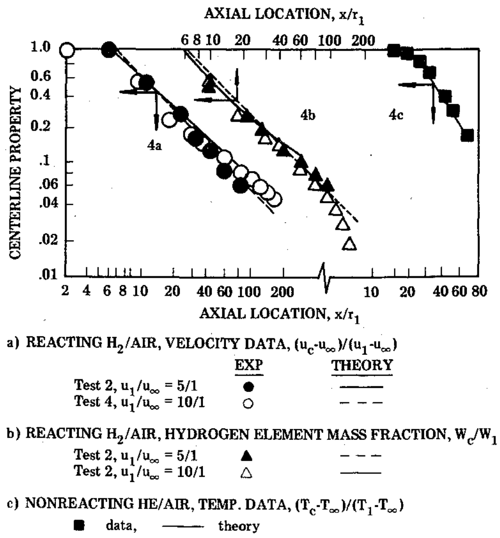

Fig. 4 Comparison of predicted and measured centerline velocity, concentration and temperature data for reacting and nonreacting jets (see Table 1 for test conditions).

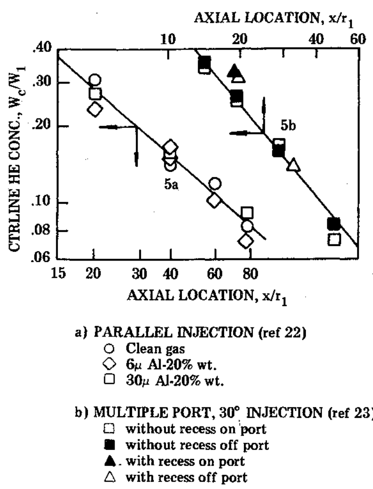

Fig. 5 Measured centerline concentration decay for particle-laden, coaxial jets.

4 and tabulated in Table 1. This measured core length has been used, together with model predictions which were made using the jet models outlined by Smoot and Purcell[8] and Smoot et. al,[9] for free and confined jets, to deduce an experimental mixing coefficient K for each experiment. These models use Eqs. (1) and (2) for the eddy diffusivity and the required value of K that predicts the observed core length is deduced. This resultant experimental K value then has been correlated as a function of jet test parameters.

Correlations for Free Submerged Jets

Donaldson and Gray[7] have shown, for free submerged jets, that the mixing coefficient is a function of the Mach number M_5. Further, those authors showed little difference in the mixing coefficients evaluated from the core-region or developed-region data. Their correlation was based on 19 velocity experiments from four investigations, including their own work. This correlation has been extended to include 23 additional velocity experiments from eight additional investigators and is shown in Fig. 6. Separate correlative lines are shown for blunt and sharp nozzles, after the manner of Donaldson and Gray. Only the K values for the core region were correlated in this study. However, developed-region K values reported by Donaldson and Gray also are shown in Fig. 6. The average difference between the correlative line and the experimentally deduced mixing coefficient was only 6.6% for all velocity experiments, whereas the maximum error was 21%.

A corresponding correlation has been developed for combined temperature and concentration decay data for free, submerged jets, based on the work of five investigators with a total of 14 experiments. This correlation is shown in Fig. 7. No distinction was made for sharp or blunt nozzles. Although the data show more scatter, and are not so extensive as those for velocity, the general trends are the same as for velocity. However, for a given Mach number, the K value for concentration/temperature is greater than that for velocity. It also has been shown[2] that

$$Sc^t = K_v/K_c; \qquad Pr^t = K_v/K_t \tag{3}$$

Hence, turbulent Schmidt or Prandtl numbers can be deduced from these two correlations. This work makes no distinction in Sc^t or Pr^t, since temperature and concentration profile data were correlated as a unit. From Figs. 6 and 7, Sc^t (or Pr^t) values increase from 0.85 to near unity as the Mach number increases. Scatter in the temperature/concentration data leads to some uncertainty in these values.

Correlations for Coflowing Coaxial Jets

Data and Correlations

The author and co-workers previously reported correlations for the turbulent mixing coefficient for coflowing coaxial jets.[2,11,36] However, since the earliest publication by Tufts and Smoot, additional data were included in the correlation. Further, the initial correlation required the evaluation of a complex parameter. Stowell and Smoot correlated the additional data and simplified the result. However, their result did not apply well to submerged jets. A new correlation is presented in this study that is based on the residual difference between the mixing coefficient and that for the submerged jet (i.e., $K-K_0$). A total of 69 velocity experiments from 12 investigations and 64 concentration/temperature experiments from seven investigations have been correlated. All of these data were for gaseous, nonreacting jets. Table 2 summarzies the data correlated. A detailed tabulation of these data is given by Stowell and Smoot.[2]

Final correlations for velocity and for temperature/concentration are given by

$$K_v - K_{vo} = 0.66\, M_5^{0.31} (\rho_1/\rho_5)^{-0.80} (u_\infty / |u_1 - u_\infty|)^{0.48} \tag{4}$$

$$K_c - K_{co} = 0.66\, M_5^{0.32} (\rho_1/\rho_5)^{-1.18} (u_\infty / |u_1 - u_\infty|)^{0.385} \tag{5}$$

where K_v and K_c are the turbulent mixing coefficients for coflowing jets, for velocity and for temperature /concentration, respectively; K_{vo} and K_{co} are the turbulent mixing coefficients for submerged jets, and numerical values are obtained from Figs. 6 and 7, respectively; M_5 is the Mach number evaluated at the radial location r_5, where $(u_c+u_\infty)/2=u_5$; ρ_1 is the primary jet gas density and ρ_5 is the gas density at r_5; u_1 and u_∞ are the primary and secondary jet velocities, respectively.

Table 3 summarizes the average error for each of these correlations, applied to various types of data and to the data as a whole. The average error for each experiment was determined by comparing the experimental K value with the value from Eqs. (4) or (5). Coaxial, submerged data also are included in this summary. K_{vo} and K_{co} values were obtained from the correlative lines of Figs. 6 and 7 and compared with experimental values. Accuracy is better for submerged jets that for coflowing jets. Accuracy is comparable for supersonic or subsonic jets, and for velocity and concentration/temperature data. For the entire set of 189 experiments, the average percent error is approximately 13.5%, which is considerably better than for previous correlations reported by the author and co-workers.

The turbulent Schmidt number (or Prandtl number) can be deduced from the ratio of Eqs. (4) and (5), since $Sc^t = K_v/K_c$. These correlations make no distinction in the turbulent

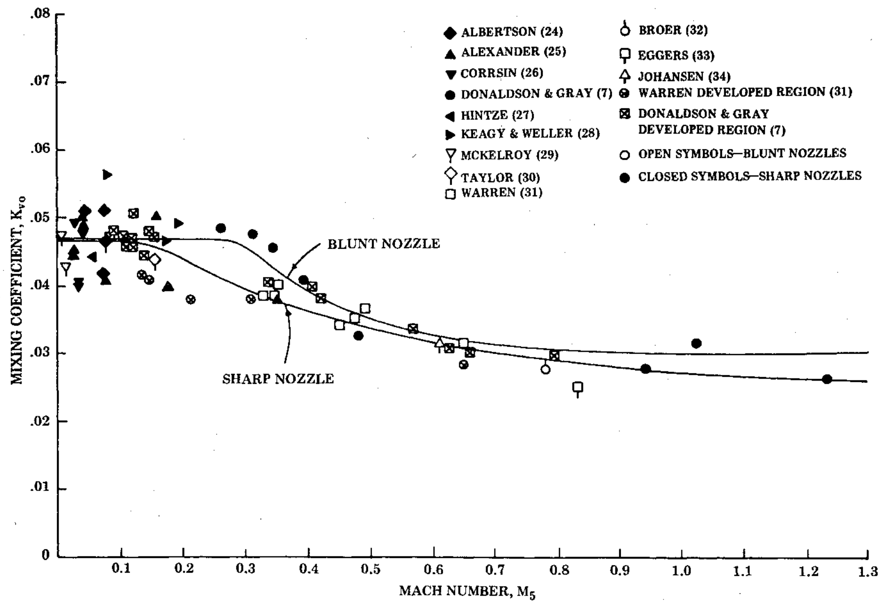

Fig. 6 Effect of Mach number on the velocity mixing coefficient for submerged jets with blunt and sharp nozzles.

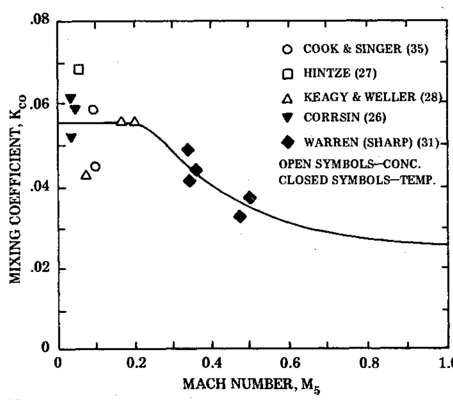

Fig. 7 Effect of Mach number on the temperature/concentration mixing coefficient for submerged jets with sharp and blunt nozzles.

Schmidt and Prandtl numbers, since the 53 concentration and 11 temperature experiments were correlated as a unit. Predicted Sc' values were not evaluated specifically for each experiment but are a function of M_5, ρ_1/ρ_5, u_∞, and u_1. For 58 experiments (Table 2), velocity and concentration or temperature measurements were made simultaneously. The average Sc' value for all of these experiments was 0.85 whereas only seven experimental values were above 0.95, with a comparable number below 0.75.

To within the accuracy of the correlations, the predicted mixing coefficient, when combined with a jet model prediction, yields the measured value of the core length. This in itself provides an extensive comparison of measured core lengths and predicted core lengths for 133 experiments. Since the logarithmic rate of decay (m) is not a strong function of K, comparison of only predicted and measured core length provides a good evaluation of the K correlation. Additional comparisons of the predicted and measured centerline decay of velocity and temperature are shown in Figs. 2-4, where agreement for the six experiments is generally very good. These predictions were made using the jet model of Refs. 8 and 9 for free and confined jets, respectively, together with the mixing coefficients of Eqs. (4) and (5) and Figs. 6 and 7.

Application to Reacting Jet Data

The correlations just discussed were developed from nonreacting jet data. However, reacting jets are of principal interest in many applications. Some studies have reported velocity, temperature, and/or concentration data for reacting gaseous jets.[20,44-47] A summary of these studies is given in Table 4.

Table 2 Summary of coflowing coaxial jet data

Author	Ref.	System	ρ_∞/ρ_1	u_∞/u_1	u_1 (m/sec)	No. of Tests Correlated		
A) Data Correlated						Velocity	Conc.	Temp.
Forstall, Shapiro	17	He-air/air	0.9	0.25-0.67	27-55	4	4	--
Landis	21	air/air	0.76-0.9	0.25-0.50	61-67	7	2	6
Landis, Shapiro	37	air/air	0.76-0.93	0.25-0.75	55-67	6	--	3
Alexander, et. al.	25	air/air	0.8	0.15	178	1	--	--
Mikhail	38	air/air	1.0	0.20	73	1	--	--
Curtet, Ricou	39	air/air	1.0	0.09-0.50	38-103	4	--	--
Fejer, et. al.	40	air/air	1.0-1.7	0.13-0.67	60-488	12	17	--
		Freon/air	1.5-2.0	0.25	72-74	2	2	--
Chriss	19	air/air	1.4	0.42	287	1	--	--
		H_2/air	0.08-0.12	0.16-0.41	582-1006	8	8	2
Brighton	41	air/air	1.0	0.10	46	1	--	--
Paulk	18	air/air	0.89	0.47	126	1	--	--
		H_2/air	.93	0.13	123	1	--	--
Peters, et. al.	42	air/air	0.81-0.94	0.12-1.52	116-126	6	6	--
Chriss, Paulk	43	air/air	0.86-0.97	0.12-1.4	120-131	6	6	--
		H_2/air	0.08-0.13	0.18-0.55	600-1000	8	8	--
Total or Range	12	4	0.08-2.0	0.09-1.52	38-1006	69	53	11

Table 3 Summary of correlation accuracy for coaxial jets

No.	Property	Data Classification	Mach Pri.	Mach Sec.	No. of Studies	No. of Tests	Range Mach No.		Mixing Coefficient Prediction Accuracy		
							M_1	M_∞	Avg.% Error	% Dev.	No. Over 30%
1	Velocity	Subsonic/Submerged	<1	0	9	35	.01-.97	0	6.7	8.5	0
2	Velocity	Supersonic/Submerged	>1	0	5	7	1.4-3.6	0	7.0	9.2	0
3	Velocity	Subsonic/Coflowing	<1	<1	12	69	.03-.90	.04-.64	14.7	17.8	8
4	Velocity	Combined Data	>0	>1	23	111	.01-3.6	.04-.64	11.7	14.9	8
5	Conc/Temp	Subsonic/Submerged	<1	0	6	14	.06-0.97	0	9.0	12.7	0
6	Conc/Temp	Subsonic/Coflowing	<1	<1	7	59	.04-0.98	.03-.73	17.1	22.2	10
7	Conc/Temp	Mixed Flows	>1	<1	2	5	1.3-1.9	.08-.35	9.5	13.6	0
8	Conc/Temp	Combined Data	>0	<1	13	78	.04-1.9	.03-.73	15.2	20.3	10

Table 4 Summary of selected experimental studies for reacting gaseous systems

Author(s)	Ref.	No. of Tests	System	Static Pres (Atm)	Velocity (m/sec)		Jet Radius Prim (cm)	Inlet Temperature (°K)	
					Prim	Sec		Prim	Sec
Anderson & Johns	44	3	air/ solid propellant	1	2.82-3.53 [a]	0	5.1-6.6	2200[b]	300
O'Connor, Comfort, Cass	45	3	N_2/air	1	1037-1143	0	1.9	5000-5850	300
Cohen & Guile	46	1	H_2/air	0.9	2075	1560	1.0	350	1750
Drewry	47	2	H_2/air	0.12	1050-1060	512-723	1.37	240	150-300
Kent & Bilger	20	4	H_2/air	1	38-180	6-24	0.38	300	300

No predictions have been made for the data of Anderson and Johns since the authors do not report the solid propellant formulation which produced the primary stream. Predictions were made for selected data from the remaining investigators. The hot N_2/air plasma jet of O'Connor et al.[45] is the only study shown with a stationary air stream surrounding the jet. In these jets, approximately 3-9% of the nitrogen was dissociated initially. O'Connor et al. verified an excellent comparison of velocity decay for one test with predictions from the Donaldson/Gray model, which is essentially the same as that shown in Fig. 6.

More recent reacting jet data are the hydrogen/air jets of Cohen and Guile,[46] Kent and Bilger,[20] and Drewry,[47] all with moving secondary air streams. Such jets provide a strong test of a predictive model since density gradients are great. Comparisons of model predictions with jet measurements of concentration, temperature, and/or velocity have been made for seven different experiments. Jet velocities range from 38 m/sec to 2073 m/sec and secondary air velocities vary from 6 m/sec to 1559 m/sec. The jet mixing model used for making these predictions was that of Smoot et al.,[28] which extended the work of Donaldson and Gray and Smoot and Purcell to include reacting systems assuming local thermodynamic equilibrium. Nonunity turbulent Prandtl and Schmidt numbers also were considered as were axial pressure gradient effects. The turbulent mixing coefficients were taken from Eqs. (4) and (5) in these predictions.

The most extensive study of reacting jets was that of Kent and Bilger.[20] Figure 4 showed comparisons of predictions and measurements for two of the experiments. More detailed comparisons are given by Smoot.[49] Primary to secondary density ratios varied from 2 to 10 in these tests. All were low-velocity subsonic tests. Agreement of measured centerline decay of velocity and concentration with model predictions for these tests was good.

Comparisons of centerline decay of concentration with predictions for the high-velocity tests of Cohen and Guile and Drewry also were reported by Smoot.[49] There is much less accuracy and detail in these data than for the data of Kent and Bilger. Although model agreement with Cohen and Guile data was reasonable, hydrogen element concentration data of Drewry were shown to decay much more rapidly than model predictions. No explanation was apparent for this lack of agreement in hydrogen decay. The core length prediction for the Drewry data was reasonable, however.

Application to Particle-Laden Jet Data

A second category of coaxial jets which were not included in the correlations reported are particle-laden jets. Tufts and Smoot[11] refer to two studies for particle-laden jets, but the measurements were not sufficiently detailed to permit inclusion in the correlations reported.

Recently, Hedman and Smoot[22] reported results of a series of confined, coflowing, coaxial jet tests using helium/air with two sizes of aluminum particles. The primary jet contained up to 20% by weight of spherical 6- or 30-μ particles whereas the secondary stream was air only. The primary jet velocity was about 290 m/sec whereas the secondary gas velocity varied from 30 to 120 m/sec. Gas composition, gas velocity, and particle concentration profiles were measured for these dusty jets. Figure 5 gave test results for 3 of these tests. These results indicate that the presence of up to 20% by weight of particles in these gas streams did not influence significantly the decay of gas composition. Somewhat larger, but still small effects of the particles were observed for the measured gas-velocity profiles. These results suggest that the mixing coefficient correlations for coaxial gaseous jets can be used for describing the gas-phase mixing in particle-laden systems, at least for these specific test conditions, and for up to 20% solids in the primary jet.

The dispersion rates of the particles in a jet are a function of initial particle lag, particle size, etc. Hedman and Smoot[22] developed a fully-coupled model for predicting particle dispersion effects, and also reported several measurements for particle dispersion rates. However, no generalized correlations are available for jet mixing coefficients for the particles.

Limitations of Mixing Coefficients for Coflowing Coaxial Jets

For submerged gaseous jets, an average error of about 10% can be anticipated, whereas for coflowing jets, an average error of 15% and a maximum error approaching 40% can be expected. The correlations reported herein also have some application to reacting jets and to particle-laden jets. The correlations are restricted in their use to the general range of test conditions shown in Table 2 and Figs. 6 and 7. Extrapolation beyond these conditions leads to additional uncertainty in the accuracy of predictions.

From Table 2, it should be observed that no experiments were correlated with u_∞/u_l values very near unity (i.e., $0.75 < u_\infty/u_l < 1.4$); further, there are only a very small number of experiments included where $u_\infty > u_l$. Prediction of jets for near-unity values of u_∞/u_l or $u_\infty > u_\infty u_l$ leads to considerably more inaccuracy using Eqs. (4) and (5) than for other jet test conditions. Also, systems with very low primary velocity (<15 m/sec, implying laminar or transition flows) were not included in the correlations.

Summary

A recorrelation of K vs M_5, for free, submerged jets has been developed including velocity data from 12 investigators and a temperature and concentration data from five investigators for this jet type. These correlations were shown in Figs. 6 and 7. Average error of these two data sets was only 6.8 and 9.0%, respectively. Further, the mixing coefficient for turbulent, coflowing, coaxial, nonreactive jets has been recorrelated. This revised result has correlated the "residual" difference between the total mixing coefficient and that for no secondary flow, for both velocity data and for temperature/concentration data. The recommended equations were shown as Eqs. (4) and (5). The average error for these two correlations, for 69 and 64 coflowing experiments was 14.7% and 17.0% for velocity and for concentration/temperature data, respectively. The accuracy of these two correlations for several subsets of data (i.e., subsonic/submerged, supersonic/submerged, subsonic/subsonic, and mixed flows) was evaluated separately. The turbulent mixing coefficient correlations can be used with a jet mixing model to predict the jet structure (mean properties) for submerged jets and coflowing jets. Although the correlations were developed from nonreacting gaseous jet data, the correlations also have been applied to reacting jets and some particle-laden jets with generally acceptable accuracy.

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